

Prove the derivative of $\cos^{-1} x$. Show all steps and justifications.

SCORE: ____ / 4 PTS

$$\begin{aligned} y &= \cos^{-1} x \longrightarrow y \in [0, \pi] \longrightarrow \sin y \geq 0 \quad (1) \\ \cos y &= x \quad (1) & \text{so } \sin y &= \sqrt{1 - \cos^2 y} \quad (1/2) \\ & & &= \sqrt{1 - x^2} \\ -\sin y \frac{dy}{dx} &= 0 \quad (1/2) \\ \frac{dy}{dx} &= -\frac{1}{\sin y} \quad (1/2) = -\frac{1}{\sqrt{1 - x^2}} \quad (1/2) \end{aligned}$$

If $f(x) = (\csc x)^{x^4}$, find $f'(x)$.

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$$y = (\csc x)^{x^4}$$

$$\ln y = x^4 \ln \csc x \quad \textcircled{1\frac{1}{2}}$$

$$\frac{1}{y} \frac{dy}{dx} = 4x^3 \ln \csc x + x^4 \frac{1}{\csc x} (-\csc x \cot x) \quad \textcircled{2\frac{1}{2}}$$

$$\begin{aligned} \textcircled{1} \frac{dy}{dx} &= (\csc x)^{x^4} (4x^3 \ln \csc x - x^4 \cot x) \quad \textcircled{1} \\ &= x^3 (\csc x)^{x^4} (4 \ln \csc x - x \cot x) \end{aligned}$$

The table shows values of $f(x)$ and $f'(x)$ at various input values. If $g(x) = \frac{f(x^3)}{x}$, find $g'(2)$.

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x	0	1	2	3	4	5	6	7	8	9
$f(x)$	-3	-1	0	2	1	4	6	5	2	-2
$f'(x)$	3	2	4	0	5	1	-1	-3	-5	-6

$$g'(x) = \frac{f'(x^3) \cdot 3x^2 \cdot x - f(x^3)}{x^2}$$

$(2\frac{1}{2})$

THIS PRODUCT CAN BE WRITTEN DIFFERENTLY, AS LONG AS IT EQUALS -120

$$g'(2) = \frac{f'(8) \cdot 12 \cdot 2 - f(8)}{4} = \frac{(-5)24 - 2}{4} = \frac{-61}{2}$$

IF YOUR FINAL ANSWER IS CORRECT, GIVE $\rightarrow$ ①

①
2

YOURSELF THIS POINT AUTOMATICALLY

If $f(x) = \ln(\sqrt{x^2 + 1} - x)$, find $f'(x)$. NOTE: In the final simplified answer, x only appears once.

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$$f'(x) = \frac{1}{\sqrt{x^2+1} - x} \cdot \left(\frac{1}{2\sqrt{x^2+1}} \cdot 2x - 1 \right)$$

$$= \textcircled{1} \left[\frac{1}{\sqrt{x^2+1} - x} \left(\frac{x}{\sqrt{x^2+1}} - 1 \right) \right] \textcircled{2}$$

$$= \frac{1}{\sqrt{x^2+1} - x} \left[\frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}} \right] \textcircled{1}$$

$$= \left[\frac{-1}{\sqrt{x^2+1}} \right] \textcircled{1}$$

Find $\frac{dy}{dx}$ if $\sin \frac{x}{y} = x^3 y^2$.

SCORE: ____ / 6 PTS

$$\boxed{\underbrace{\left(\cos \frac{x}{y}\right)}_{\textcircled{1}} \underbrace{\left(y - \frac{xy'}{y^2}\right)}_{\textcircled{1}}} = \underbrace{3x^2 y^2}_{\textcircled{1}} + \underbrace{2x^3 y y'}_{\textcircled{1}}$$

$$\left(\cos \frac{x}{y}\right) \left(y - \frac{xy'}{y^2}\right) = 3x^2 y^4 + 2x^3 y^3 y'$$

$$y \cos \frac{x}{y} - \left(x \cos \frac{x}{y}\right) y' = 3x^2 y^4 + 2x^3 y^3 y'$$

$$\boxed{y \cos \frac{x}{y} - 3x^2 y^4} = \boxed{2x^3 y^3 y' + \left(x \cos \frac{x}{y}\right) y'}_{\textcircled{1}}$$

$$\frac{dy}{dx} = \boxed{\frac{y \cos \frac{x}{y} - 3x^2 y^4}{2x^3 y^3 + x \cos \frac{x}{y}}}_{\textcircled{1}}$$

TALK TO ME IF
YOU REWRROTE
THE EQUATION
FIRST

If the position function of an object is given by $f(t) = \tan^{-1} t^3$, find the acceleration function.

SCORE: ____ / 5 PTS

$$f'(t) = \frac{1}{1+(t^3)^2} 3t^2 = \boxed{\frac{3t^2}{1+t^6}} \textcircled{2}$$

$$f''(t) = \boxed{\frac{6t(1+t^6) - 3t^2(6t^5)}{(1+t^6)^2}} \textcircled{2}$$

$$= \frac{6t + 6t^7 - 18t^7}{(1+t^6)^2}$$

$$\textcircled{1} \boxed{\frac{6t - 12t^7}{(1+t^6)^2}} = \frac{6t(1-2t^6)}{(1+t^6)^2}$$